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The Radar Logic Daily™ Index

By: Marios Kagarlis[a], David Peterson[b], Robert Eberlein[c], and Thomas Fiddaman[d]

[a] Dr. Kagarlis (PhD Physics, U. Penn.) is Chief Scientist of Ventana® and lead inventor of the Real Estate Daily Index patent pending and other patents of agent simulators.

[b] Dr. Peterson (PhD EE, MIT) is Chairman of Ventana and holds patents enabling detection and prevention of data errors and rapid comprehension of results.

[c] Dr. Eberlein (PhD Mgt., MIT) is Chief Software Architect at Ventana.

[d] Dr. Fiddaman (PhD System Dynamics, MIT) from Ventana does work in energy, economics and the environment and won the 2006 Jay Forrester award for his work on Environment-Economy interactions.

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Abstract

In the following we describe the Radar Logic Daily Index for residential real estate transactions. We review the objective for real estate indexing and give a brief overview of our index and its advantages. We present the probability density function upon which we base the index, the Triple Power Law™ (TPL), chosen for accurate representation of the empirical spectra of price per square foot in residential real estate transactions. We explain how we fit the TPL parameterization to the data using maximum likelihood estimation and optimization techniques. We conclude with the specification of the index and a summary of its advantages.

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1. The Need for Real Estate Indexing

Real estate is a major asset class without a robust derivatives market. Without the futures and options contracts available for traditional commodities, investors lack housing-specific financial tools to manage risk and capture opportunity. The goal of a real estate index is to provide in a usable and reliable manner a way to express the value of real estate through time and across geographies. Efforts to analyze information on real estate prices have been plagued by limitations:

- Data is available infrequently, tending to be published monthly, quarterly or semi-annually.
- Sales transaction volumes fluctuate widely over time, may be subject to seasonal effects, and vary greatly across geographical areas.
- Each property is unique, making it difficult to compare individual properties within a market and even harder to compare from one geographic area to another.
- Public source records have inconsistencies due to the many local jurisdictions involved and their varying data processing standards.

The methodology developed for the computation of the Radar Logic Daily Index is designed to produce an accurate benchmark suited for creating and settling financial derivatives despite these limitations.

2. Overview of Triple Power Law and the Radar Logic Daily Index

Five objectives have guided the development of the Radar Logic Daily Index. It must:

1. accurately capture the <u>daily</u> movement of values in the real estate market;
2. use all the available data;
3. remain robust in the face of abrupt changes in market conditions;
4. give reliable results for low-volume days with sparse, scattered transactions; and
5. maintain reliability in the presence of error, manipulation and statistical outliers.

In order to achieve the above objectives we systematically have analyzed home sales transactions representing major Metropolitan Statistical Areas (MSA's) in the United States and spanning years of historical data. We have explored attributes published in connection with real estate transactions or derived from them. We have tested a wide range of statistical measures and weightings as potential candidates for computing the index. The following overview summarizes the logic leading to our index; subsequent sections flesh out the technical details.

To characterize the real estate transactions occurring in an area, we need a measure that will allow us to compare small and large homes. Simply looking at the prices at which an existing house changes hands is limited by the information it ignores. Further, the uniformity of the asset value is not guaranteed as renovations may have occurred; the length of time between transactions is variable; and there is no possibility of including new home sales.

Rather than looking at the price of a house, we look at the price per square foot (ppsf) of a house as a means to make transactions comparable. (This is the accepted practice in commercial real estate, is also used by most builders, and, less formally, by those in the market for a new home.) From a trading perspective this makes the transactions more similar, but unlike a more fungible commodity such as oil, there are still significant differences among houses.

Therefore for real estate, we need to characterize a distribution of transactions. When we look at the real estate transactions occurring on a given day we see a distribution of prices, even when measured per square foot. This is because there are many attributes besides square footage that determine the value of a house. Thus, in deriving an index for real estate, we must use a composition of prices from all the different properties for which transactions are recorded. Creating that composition must be done with care, as many simple measures may not accurately reflect true market value.

We probed extensively for recognizable patterns in the distribution of daily ppsf distributions and found strong empirical evidence that residential real estate transactions in large metropolitan markets are described by power laws (i.e., over an interval, the frequency of transactions is proportional to the ppsf raised to a power). The data reveal power law behavior with three distinct power laws in the low, middle and high ends of the price spectrum. The specific price range of each sector and its composition in types of properties varies with geography and over time

We have named this three-component distribution the Triple Power Law (TPL for short). We apply the TPL to daily sales transactions. The result of this process is to encapsulate an entire distribution of price-per-square-foot transactions into a single mathematical distribution from which a reliable and representative single index can be deduced.

Additionally, we have validated systematically that the *shape* and *position* of the ppsf distribution are determined by distinct dynamics with different characteristic timescales.

- The shape conveys the distribution of *relative quality* of the housing stock in a given market. This shape is stable in the short term, changing slowly over time in a manner that reflects longer-term socioeconomic and cultural trends.
- The position reveals the correspondence between quality and *value* in the local market on a given day, as determined by that day's actual sales. This value is susceptible to short-term shifts in the economy, changes

in market sentiment, and news shocks, and can be volatile even as the underlying housing stock remains unaltered.

Using the discovery that TPL describes succinctly and accurately the empirical ppsf distribution, and further, representing the overall range of market quality with the shape and the daily value with the position of the probability distribution function (PDF), we now present the methodology to formulate a consistent index.

Here we encapsulate TPL in a parameterization that we then “fit” to the actual housing sales data. Each day yields a set of parameters, and the closer the fit to the actual data, the higher the confidence that the resulting index value accurately reflects the marketplace. The fitting algorithm to find the closest matching set of parameters is a non-linear search over the parameter space which includes the methodical variation of the parameter values, the determination at each step whether improvement has been achieved, and a termination criterion for deciding that maximum convergence has been attained between the model PDF and the actual data.

We devised a mathematical form for the parameterization which exhibits power laws over three distinct ranges of ppsf and conveniently separates out the shape from the position dependence of the distribution so as to allow accounting for their respective timescales. This separation has several benefits.

First, in our parameterization the parameters that capture the overall shape of a market’s ppsf distribution are the most numerous. Since the shape is stable in the short term and the parameters that describe it have been disentangled from the more volatile position, their computation can use data collected over a longer time period. The resulting higher volume of sales transactions boosts the quality of the fit and the statistical confidence in the TPL shape as an accurate snapshot of how quality is distributed in the local housing stock.

Second, some geographical areas exhibit notable periodicity in transaction volume and ppsf (e.g. Boston houses sell more slowly and for less in the winter). Being able to use data over a longer time period for the shape parameters allows incorporating a full annual cycle, ensuring that seasonal effects do not introduce artificial distortions in the derived shape. Therefore we have chosen to use a year’s worth of data as the relevant timescale for computing the shape parameters – more precisely the workdays among the three hundred sixty five calendar days up to the date for which the index is computed, a distinction which stresses that there is no aggregation but the data are kept separate for each workday.

Perhaps most importantly, the third benefit from formulating TPL so as to disentangle the shape from the position dependence is that the latter is reduced to a single parameter. This is crucial since the daily transaction volume can be so low as to potentially induce a multi-parameter fit that depends exclusively on it to yield low-confidence values. Capturing the volatility of the market's movement in a single parameter essentially enables a daily index, ensuring that a day's transaction volume even if low is adequate to fix the position of the ppsf spectrum to within statistical uncertainty compatible with the actual data.

To reiterate, data over the preceding full year are used for the computation of each day's shape parameters, while only the actual sales transaction data for that day are used for the computation of its position parameter. A sketch of the fitting procedure that implements the above scheme follows:

1. For a metropolitan statistical area (MSA) and time interval of interest, a loop is entered over all the workdays for which the index is to be computed.
2. For each workday, the shape parameters are simultaneously varied and fixed for all the intermediate workdays in the calendar year leading up to the current workday.
3. For each set of shape parameters, a loop is entered over the intermediate workdays of the preceding calendar year up to the current date. For each intermediate workday the position parameter is varied separately and its respective *likelihood function* is computed. A likelihood function is a standard statistical construct which, used in conjunction with a model PDF of a variable it purports to be describing, conveys how likely it is for a given empirical spectrum of that variable to have been generated by the model PDF. A likelihood function comprises a product of terms, each of which is the value of the model PDF evaluated at each point in the dataset. For TPL the underlying variable is ppsf. We use a variant, the log likelihood function, which comprises the sum of logarithms of terms as described above instead of their product. This avoids numerical instabilities and facilitates more reliable fits.
4. The search for an optimum position parameter, given a set of shape parameters, eventually converges for each intermediate workday. When this happens the fitting algorithm returns the value of the position parameter that maximizes that day's log likelihood function, together with the value of the latter.
5. Once step (4) has been completed for each intermediate workday, the cumulative log likelihood function for all the intermediate workdays of the preceding year up to the current day is computed as the sum of the respective maximized log likelihood values of all the intermediate workdays. The fitting algorithm then determines whether further maximization of the cumulative log likelihood function is possible, in which case it iterates steps (2 – 5); otherwise the shape parameter search is terminated.
6. On terminating, a set of values has been obtained that renders an initially abstract TPL parameterization into an empirical PDF that describes accurately the data for the current workday. The index for the current workday is derived from this PDF.
7. Steps (1 – 6) are iterated for all the workdays and MSA's of interest.

After exploring numerous candidates for the index, we have concluded that the median derived from TPL is the most representative and robust, exhibiting the least sensitivity to data errors or manipulations, outliers, and low transaction volumes. We have therefore chosen the TPL-derived median as the Radar Logic Daily Index.

3. Background on Power Laws

Found in science, economics and the social sciences, power laws express a relationship between two variables that exhibits scale invariance, or simply put is preserved at both small and large scales. Mathematically, two quantities x, y are related by a power law with exponent β and proportionality constant a if one is proportional to a power of the other, as in the following equation:

$$y = a\,x^{\beta} \tag{3.1}$$

If x, y represent a pair of values of two quantities related via a power law (3.1), and x', y' another pair of values of the same two quantities also obeying the same power law, the two pairs of values are related via

$$\frac{y}{y'} = \left(\frac{x}{x'}\right)^{\beta} \tag{3.2}$$

which removes the proportionality constant a. In logarithmic scale, the relationship (3.2) becomes

$$\log y = \log y' + \beta\left(\log x - \log x'\right) \tag{3.3}$$

which is a straight line equation relating the logarithms of the quantities in (3.2).

There is little dispute that power laws do describe empirically a large number of real-world phenomena, as for example Zipf's Law (the frequency at which words are used in a language as a function of their complexity) and Pareto's Law (the "80/20" distribution of wealth) to name two[†]. In real estate, power law behavior has been noted in the distribution of land prices in Japan, and of urban real estate prices in Sweden. It is plausible that the often-observed power law

[†] Zipf's and Pareto's laws represent a somewhat different manifestation of power laws than the one we use, probing distributions of ranks derived from the cumulative distribution function of a variable. We probe the probability density function of the variable itself, resulting in a manifestation of power laws more common in the natural sciences. The two formulations are in principle equivalent and can be recast into each other.

distribution of wealth would be reflected in a power-law distribution of housing values.

Ultimately, however, our decision to invoke TPL for the formulation of our index was taken on purely pragmatic grounds: we have unambiguously observed power laws in the ppsf spectra of voluminous historical housing data. The residential real estate market price per square foot distributions obey power laws.

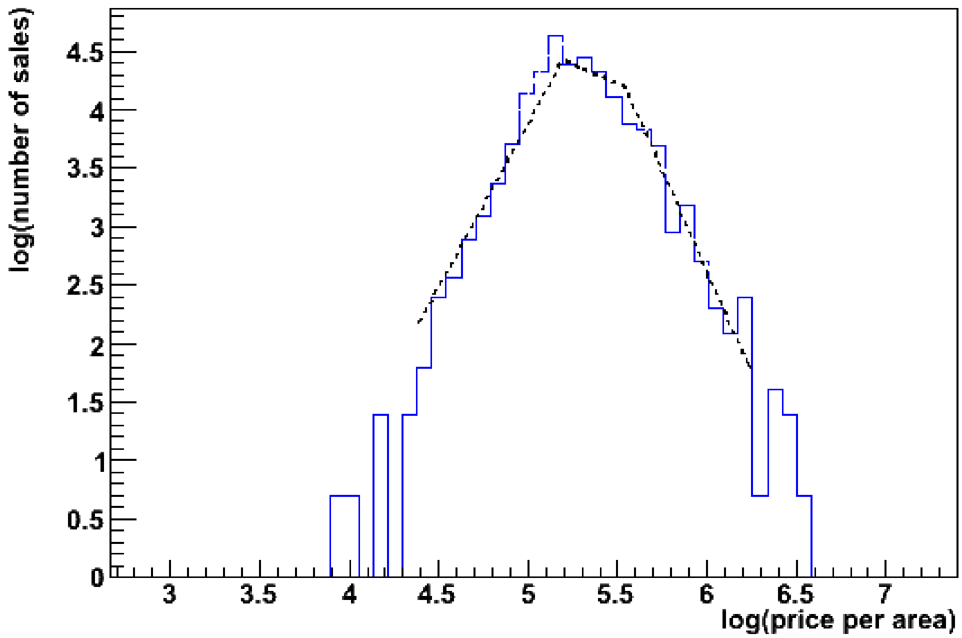
4. Evidence for the Triple Power Law

In the case of residential real estate transactions, if a ppsf value and its frequency of occurrence (i.e., the number of sales per ppsf value) are related by a power law, then that power law can be obtained by replacing x, y in Equation (3.3) respectively by ppsf and N the number of home sales per given ppsf value:

$$\log N = \log N' + \beta\left(\log ppsf - \log ppsf'\right) \tag{4.1}$$

In rendering the ppsf spectra as histograms, the height of each bin represents the number of sales with ppsf values within the range covered by that bin. The last statement implies that we attribute the same importance to each sale; in mathematical terms this means that we use unweighted ppsf values, or equivalently attribute a weight factor of 1 to each sale. After experimenting with other weightings, including area and price, we concluded that they introduce noise and amplify volatility, so we have opted against them. With the substitutions leading to Equation (4.1) it follows that if ppsf and N obey a power law, displaying a ppsf histogram which represents a daily transaction spectrum in log-log scale ought to reveal straight lines in each of three regions, corresponding to the low, middle and high end of the housing market. In general this is the case.

Figure 1: A typical daily ppsf spectrum in log-log scale with fixed size bins for the home sales recorded on a specific date for a metropolitan area. The spectrum exhibits three approximate straight-line regions shown by the dashed line as obtained from the TPL fit, corresponding to power laws with distinct exponents β.

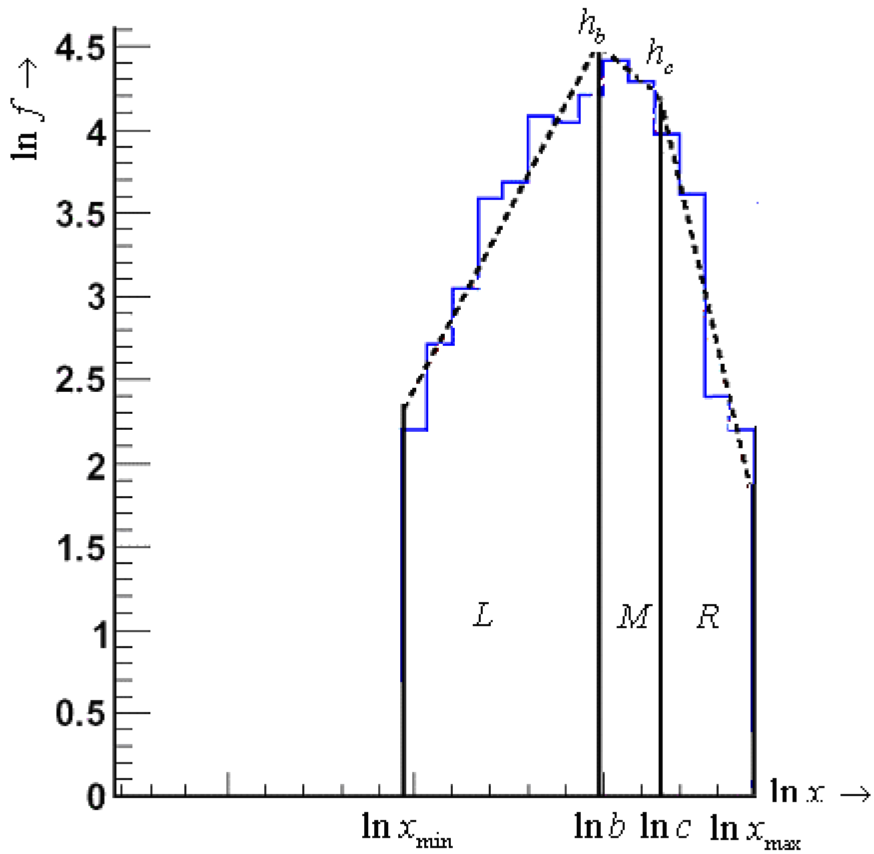


5. The Triple Power Law Formulation

5.1 Parameterization

Referring to Figure 2, below, we show a hypothetical *ppsf* spectrum that illustrates the TPL parameterization. The spectrum is presented as a histogram in logarithmic scale in both the abscissa x, which denotes *ppsf* for brevity, and the ordinate f that stands for the frequency. For presentation purposes we use bins of fixed size in $\ln x$ as this brings the power-law behavior more clearly to the fore. As shown in the figure, $x_{\min}, x_{\max}$ denote respectively the lowest and highest *ppsf* values of the hypothetical dataset. For purposes of computation we fix the values of these cutoffs to $x_{\min} = 10^{-5}, x_{\max} = 10^{6}$ respectively, so that all *ppsf* values encountered in daily datasets are accepted. Let L (left), M (middle) and R (right) denote the three regions of TPL over which distinct power laws apply, with respective ranges $(x_{\min}, b)$, (b, c), $(c, x_{\max})$, and exponents $\beta_{L,M,R}$. Let $h_{b,c}$ be the natural logarithms of the frequency respectively at b, c. In log-log scale the exponents of power laws appear as slopes of line segments. We note in passing that, as an artifact of using a fixed bin size in logarithmic scale, the slopes in the histogram of Figure 2 are augmented by 1 relative to the true exponents $\beta_{L,M,R}$ of the respective power laws.

Figure 2: Illustration of the Triple Power Law parameterization (dashed line) superposed on a hypothetical dataset of ppsf values displayed as a histogram, with the natural logarithms of *ppsf* as abscissa and of the frequency as ordinate. We use a fixed bin size and notation as explained in the text.



Our goal is to derive a probability density function consistent with TPL for datasets of home sales in a given date and location that resemble the spectrum of Figure 2. To do so we write down expressions similar to Equation (3.2) for each of the regions L, M and R.

$$f(x) = \begin{cases} h_b \left(\frac{x}{b} \right)^{\beta_L}; & x_{\min} \le x \le b \\ h_c \left(\frac{x}{c} \right)^{\beta_M}; & b < x \le c \\ h_c \left(\frac{x}{c} \right)^{\beta_R}; & c < x \le x_{\max} \end{cases} \quad (5.1)$$

The effective cutoffs, as noted earlier, are fixed to $x_{\min} = 10^{-5}, x_{\max} = 10^{6}$. The low cutoff coincides with the error threshold for single-precision computations and the upper cutoff exceeds any possible *ppsf* value that can be expected in genuine real-estate transactions. Hence these cutoffs ensure that no valid data are rejected.

We note that the parameterization (5.1) already matches the two power laws in the middle and right regions at their interface c. This constraint is necessary for physical behavior, since there can be no discontinuities in the distribution as *ppsf* approaches the boundary between two adjacent regions from the left or from the right. We need however to also enforce this physical requirement at the interface b between the left and middle regions. To do so we evaluate the power law equation for the middle region at b, and require that its value there matches h_b, which is the value of the power law on the left at that point. As a result of imposing this constraint the slope of the power law in the middle region becomes fixed:

$$h_c \left(\frac{b}{c} \right)^{\beta_M} = h_b \Rightarrow \beta_M = \frac{\ln h_c - \ln h_b}{\ln c - \ln b} \tag{5.2}$$

Hence, as a consequence of imposing a necessary physical constraint on Equation (5.1), we have also reduced by one the number of parameters remaining to be fixed by the fit.

We next note that the function $f(x)$ of Equation (5.1) must be normalized to unity in order for it to be a valid PDF. To illustrate what this requirement means, we paraphrase it as an equivalent statement: if one picks at random the *ppsf* value of a transaction in a daily dataset, that value is certain (i.e. has probability 1) to lie below and up to the maximum value in the dataset. Although this statement is self evident, it has to be imposed mathematically on the TPL parameterization. As written in Equation (5.1), TPL exhibits a desired power law behavior which qualitatively matches that of the empirical ppsf spectra, but it is not yet properly normalized. Formally, this is achieved by forcing the integral of the PDF over its entire range to be unity:

$$I \equiv \int_{x_{\min}}^{x_{\max}} dx\, f(x) = 1 \tag{5.3}$$

Before proceeding to the evaluation of this integral we make a couple of convenient parameter substitutions. Since we have not yet normalized $f(x)$ of Equation (5.1), its absolute scale is arbitrary up to an overall multiplicative constant. We take advantage of this and let for convenience

$$h_b = 1 \tag{5.4}$$

introducing at the same time an overall scale parameter s which multiplies $f(x)$.

For reasons that will become evident shortly, we would also like to eliminate the explicit dependence of β_M on b in the denominator of the right-hand side of Equation (5.2). To do so we introduce an auxiliary parameter p by means of which we express c as a multiple of b, noting that because of our definition of the three regions we must have $b \le c$. We can then recast c as follows:

$$c = pb;\ 1 < p \tag{5.5}$$

In effect, what we have done is to replace the search over parameter c by a search over parameter p given a value for b, with the constraint that p has to be greater than 1.

With the above substitutions the expression for β_M of Equation (5.2) reduces to:

$$\beta_M = \frac{\ln h_c}{\ln p} \tag{5.6}$$

In order for the shape of TPL to be physical, the conditions $\beta_L > 0,\ \beta_R < 0$ must be satisfied. Though in principle these allow for $\beta_{M,R} = -1$, for the historical data we have analyzed they fall more steeply, or $\beta_{M,R} < -1$. Noting that I in Equation (5.3) is the sum of three integrals over Regions L, M and R of Figure 2, for $\beta_{M,R} \ne -1$ we have:

$$\begin{aligned}
I_L &= sI'_L; \quad I'_L = \frac{b}{\beta_L + 1}\left[1 - \left(\frac{x_{\min}}{b}\right)^{\beta_L + 1}\right] \\
I_M &= sI'_M; \quad I'_M = \frac{bph_c}{\beta_M + 1}\left[1 - \frac{1}{p^{\beta_M + 1}}\right] \\
I_R &= sI'_R; \quad I'_R = \frac{bph_c}{\beta_R + 1}\left[\left(\frac{x_{\max}}{bp}\right)^{\beta_R + 1} - 1\right] \\
I &= s\left(I'_L + I'_M + I'_R\right)
\end{aligned} \tag{5.7}$$

For completeness we show the corresponding equations for $I'_{M,R}$ if either $\beta_{M,R} = -1$, which replace the respective expressions in (5.7):

$$
\begin{aligned}
I'_M &= bph_c \ln p \\
I'_R &= bph_c \left(\ln x_{\max} - \ln (bp) \right)
\end{aligned}
\tag{5.8}
$$

Since s is an overall constant which multiplies all three integrals of Equation (5.7), the normalization condition $I = 1$ can be achieved easily by setting:

$$s = 1 \big/ \left(I'_L + I'_M + I'_R \right) \tag{5.9}$$

This fixes the scale s and turns $f(x)$ into a proper PDF consistent with TPL.

We recap all of the above by recasting the TPL parameterization as

$$
f(x) = s \begin{cases}
x'^{\beta_L}; & x'_{\min} < x' \le 1 \\
h_c \left(\dfrac{x'}{p} \right)^{\beta_M}; & 1 < x' < p \\
h_c \left(\dfrac{x'}{p} \right)^{\beta_R}; & p \le x' \le x'_{\max}
\end{cases}
\tag{5.10}
$$

where

$$x' = x/b,\ \ x'_{\min} = x_{\min}/b,\ \ x'_{\max} = x_{\max}/b \tag{5.11}$$

The motivation for the substitution (5.5) was to enable disentangling the shape from the position of the TPL distribution in logarithmic scale, achieved in Equations (5.10) and (5.11), with the shape captured in $p, h_c, \beta_{L,R}$ and the position in the single parameter b.

5.2 Maximum Likelihood Estimation

Maximum Likelihood Estimation is a powerful statistical method which enables finding values for a parameterization of a model PDF such as to maximize the likelihood that a given data sample has been produced by the hypothesized PDF. This is a common fitting algorithm, broadly used in science and engineering. This method requires a theoretical PDF, such as the TPL. The likelihood estimation is sensitive to the model PDF chosen to represent the empirical data, and the ability to achieve good fits compatible with the statistical uncertainty inherent in the data corroborates that the PDF model is sensible.

To fix the remaining parameters $p, h_c, \beta_{L,R}, b$, we build the log likelihood function, as described in the Overview, by taking the sum of the natural logarithms of $f(x)$ evaluated at each $ppsf$ value in a given dataset. The log likelihood function for a given daily dataset is:

$$LL = \sum_{i=1}^{N} \ln f(x_i) \qquad (5.12)$$

where x_i are the actual $ppsf$ values of individual sales i in a dataset of N sales.

Fitting for the remaining parameters entails maximizing LL, which can be achieved by using one of several standard fitting algorithms documented extensively in the science and engineering literature. At present we use Powell's method, a non-critical choice for our purposes.

5.3 Fitting Procedure

Fitting multi-parameter functions can present challenges for low-volume datasets, such as those found on some days and geographies. We have addressed this issue as satisfactorily as is possible by capturing the volatile aspects of the $ppsf$ distribution, namely its position, into a single variable. This allows using data over a considerable length of time to fit for the four shape parameters, which yields high-confidence values, and limiting the sensitivity to statistical fluctuations due to low daily volumes to the single position parameter.

The two-step shape and position search, explained in the Overview in some detail, is summarized below:

- For each workday we implement a two-step fitting algorithm in which:
  - The shape parameters $p, \beta_{L,R}, h_c$ are fixed simultaneously for all the workdays of the 365 calendar days leading up to the current date, and optimized in an outer call to the fitting algorithm, which maximizes $\sum_{i=current\,date}^{current\,date-365} \begin{cases} LL_i; & i \text{ is tradable workday} \\ 0; & \text{otherwise} \end{cases}$.
  - The position parameter b is optimized for each intermediate workday of the 365 calendar days, by maximizing each respective LL_i independently via inner calls to the fitting algorithm.
  - The optimized values are retained and attributed to the current date.
  - This procedure is iterated for each date of interest.

The outcome of this is optimized values for all the parameters of the day's PDF.

6. The Radar Logic Daily Index

Equipped with a daily set of values for the TPL parameters, $f(x)$ is transformed into an empirical PDF for each day. Let us denote the TPL-derived median as $\tilde{x}$. By definition, if $\tilde{x}$ represents the median *ppsf* of a dataset of home sale transactions, picking a random transaction in that dataset has a 50-50% probability to be higher or lower respectively of the median. Formally, this translates into the mathematical statement that the integral of the PDF up to the median yields the value 1/2:

$$\int_{x_{\min}}^{\tilde{x}} dx \; f(x) = \frac{1}{2} \tag{6.1}$$

The evaluation of the integral above depends on how the *ppsf* values in the distribution are split among the three regions L, M and R, or equivalently the values of the integrals $I_{L,M,R}$ of Equation (5.7). Specifically, depending on $I_{L,M,R}$, $\tilde{x}$ for $\beta_{M,R} \neq -1$ evaluates to

$$\tilde{x} = b \times \begin{cases} \left[\frac{1}{2}\frac{\beta_L+1}{sb} + x'^{\beta_L+1}_{\min}\right]^{\frac{1}{\beta_L+1}}; & I_L > 0.5 \\ \left[\left(\frac{1}{2} - I_L\right)\frac{\beta_M+1}{sb}\frac{p^{\beta_M}}{h_c} + 1\right]^{\frac{1}{\beta_M+1}}; & I_L + I_M > 0.5 \\ p\left[\left(\frac{1}{2} - I_L - I_M\right)\frac{\beta_R+1}{sb}\frac{1}{ph_c} + 1\right]^{\frac{1}{\beta_R+1}}; & \text{otherwise} \end{cases} \tag{6.2}$$

while for $\beta_{M,R} = -1$

$$\tilde{x} = b \times \begin{cases} \exp\left(\frac{0.5 - I_L}{sh_c bp}\right); & I_L + I_M > 0.5 \text{ and } \beta_M = -1 \\ p\exp\left(\frac{0.5 - I_L - I_M}{sh_c bp}\right); & I_L + I_M + I_R > 0.5 \text{ and } \beta_R = -1 \end{cases} \tag{6.3}$$

This concludes the derivation of the Radar Logic Daily Index.
To summarize, the Index

- is daily;
- captures and utilizes as much data as possible;
- reacts as the market moves, not in a delayed or "smoothed" fashion;
- reflects data driven values regardless of actual data volume; and
- avoids manipulation by illegitimate or erroneous data.